A CASH FLOW EOQ INVENTORY MODEL FOR NON-DETERIORATING ITEMS WITH CONSTANT DEMAND

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ABSTRACT
This study presents an inventory model to determine an optimal ordering policy for non-deteriorating items and time independent demand rate with delay in payments permitted by the supplier under inflation and time discounting, and the rate is assumed to be constant. This study determines the best cycle period and optimal payment period for items so that the annual total relevant cost is minimized. The main purpose of this study is to investigate the optimal (minimum) total present value of the costs over the time horizon \( H \) for both cases where the demand is fixed (constant) at any time. This study work is limited to only non-deteriorating goods with constant demand and with a permissible delay in payment. Numerical example and sensitivity analysis are given to evince the applicability of the model.

Keywords: Demand, Inventory, Non-Deterioration, Inflation, Delay in payments, Replenishments.

INTRODUCTION
In a continuous, or fixed-order-quantity, system when inventory reaches a specific level, referred to as the reorder point, a fixed amount is ordered. The most widely used and traditional means for determining how much to order in a continuous system is the economic order quantity (EOQ) model, also referred to as the economic lot-size model. The earliest published derivation of the basic EOQ model formula in 1915 is credited to Ford Harris, an employee at Westinghouse. The economic order quantity (EOQ) is the order quantity that minimizes total holding and ordering costs for the year. Even if all the predictions don’t hold accurately, the EOQ gives us a good indication of whether or not present order quantities are reliable (Bozarth, 2011). The economic order quantity, as mentioned, is the order size that minimizes the sum of carrying costs and ordering costs. These two costs react inversely to each other. As the order size increases, fewer orders are required, causing the ordering cost to decline, whereas the average amount of inventory on hand will increase, resulting in an increase in carrying costs. Thus, in effect, the optimal order quantity represents a compromise between these two inversely related costs.

Of recent, Ouyang et al., (2006) developed a model for non-instantaneous deteriorating items under permissible delay in payment. An EOQ model under conditionally permissible delay in payments was developed by Huang (2007). Jaggi et al., (2008) developed a model retailer’s optimal replenishment decisions with credit-linked demand under permissible delay in payments. Optimal retailer’s ordering policies in the EOQ model for deteriorating items under trade credit financing in supply chain was developed by Mahata and Mahata (2008). Musa and Sani (2009) developed an EOQ model for items that exhibit delay in deterioration. Hou and Lin (2009) developed “a Cash Flow Oriented EOQ model with delay in payment for deteriorating goods”. Chiu et al., (2010) jointly determined economic batch size and optimal number of deliveries for EPQ model with quality assurance issue. Feng et al., 2010 presents an algebraic approach adopted to re-examine Chiu et al.,’s model (2010). Tripathi et al., (2010a), obtained and ordering policy for non-deteriorating items and time dependent demand rate under inflation and time discounting. Tripathi et al., (2010b) developed EOQ model credit financing in economic ordering policies of non-deteriorating items with time-dependent demand rate in the presence of trade credit using a discounted cash-flow (DCF) approach. Tripathi et al., (2011) developed an inventory model for deteriorating items over a finite planning horizon considering the effects of inflation and permissible delay in payment. Musa and Sani (2012) also developed an EOQ model for items that exhibit delay in deterioration under permissible delay in payment. Also, Amutha et al., (2013) presented an inventory model for constant demand with shortages which are completely backlogged. There are also inventory model with permissible delay in payment for linear demand rate of non-deteriorating goods. Dari and Sani (2013) developed an EPQ model for items that exhibit delay in deterioration with reliability consideration for which the demand before and after deterioration are assumed to be constant.

In this paper, we develop an EOQ inventory model that determine an optimal ordering policy for non-deteriorating items and time independent demand rate with delay in payments permitted by the supplier under inflation and time discounting, and the rate is assumed to be constant. Thus, this paper is an extension of Tripathi et al., (2010a).

NOTATIONS
\[ Y (t) \]: Inventory at any time \( t \)
\[ Q \]: Order quantity, units/cycle
\[ H \]: Length of planning horizon
\[ L \]: Replenishment cycle time
\[ n \]: Number of replenishment during the planning horizon, \( n = \frac{H}{n} \)
\[ \frac{5}{6} \]: Rate per unit time
\[ a \]: Unit cost of the item, \( \$/unit \)
\[ h \]: Inventory holding cost per unit per unit time excluding interest charges,
\[ r \]: Discount rate represent the time value of money
\[ f \]: Inflation rate
\[ k \]: The net discount rate of inflation \( (k = r - f) \)
\[ Y_c \]: The interest earned per dollar in stocks per unit time by the
A Cash Flow EOQ Inventory Model For Non-Deteriorating Items

MATHEMATICAL FORMULATION
To develop the mathematical model, the following assumptions are being made:

- The demand $D$ is constant.
- Shortages are not allowed.
- Lead time is zero.
- The net discount rate of inflation rate is constant.

- case I: $m$ (permissible delay in settling account) $\leq L$ (Replenishment cycle time).
- case II: $m$ (permissible delay in settling account) $>L$ (Replenishment cycle time).

The inventory $Y(t)$ at any time $t$ is depleted by the effect of demand only. Thus the variation of $Y(t)$ with respect to $t$ is governed by the following differential equation

$$\frac{dY(t)}{dt} = -D \quad 0 \leq t \leq L = \frac{H}{n}$$

Solving (1), we have

suppliers

$Y_c$: The interest charged per dollar in stocks per unit time by the supplier.

$m$: The permissible delay in settling account

$ip$: Interest payable during the first replenishment cycle

$z_1(n)$: The total present value of the cost over the time horizon $H,$

for $m \leq L = \frac{H}{n}$

$z_2(n)$: The total present value of the cost for $m > L = \frac{H}{n}$

$E$: The interest earned during the first replenishment cycle

$E_1$: The present value of the total interest earned over the time horizon

$Y_p$: The total interest payable over the time horizon $H$

Fig 1: Inventory movement in a non-instantaneous deterioration situation
\[ Y(t) = -Dt + k \quad \text{(where } k \text{ is a constant)} \quad (2) \]

From fig 1, we have the initial condition \( Y(t) = Y_0 \) at \( t = 0 \). Substituting this in (2), we get
\[ Y(t) = -Dt + Y_0 \quad (3) \]

From fig 1, \( Y(L) = 0 \). Substituting in (3), we have
\[ Y_0 = DL \quad (4) \]

Substituting (4) into (3), we get
\[ Y(t) = -Dt + DL \quad (5) \]

The initial inventory \( Y_0 \) (order quantity) after replenishment is given by
\[ Q = Y(0) = DL \quad (6) \]

The total present value of the replenishment cost is given by:
\[ C_1 = \sum_{j=0}^{n-1} A_0 e^{-jL} = A_0 \left[ \frac{1 - e^{-kH}}{1 - e^{-kL}} \right] \quad (L = H / n) \quad (7) \]

The total present value of the purchasing costs is given by:
\[ C_2 = c \sum_{j=0}^{n-1} Y(0)e^{-jL} = cQ \left[ \frac{1 - e^{-kH}}{1 - e^{-kL}} \right] \quad (L = H / n) \quad (8) \]

The present value of the total holding costs over the time horizon \( H \) is given by
\[ A = h \sum_{j=0}^{n-1} e^{-jL} \int_0^L Y(t)e^{-kt} dt \]
\[ = h \sum_{j=0}^{n-1} e^{-jL} \left[ D \left( \frac{L}{k} + e^{-kL} - 1 \right) \right] \]
\[ A = h \left[ D \left( \frac{L}{k} + e^{-kL} - 1 \right) \right] \left[ \frac{1 - e^{-kH}}{1 - e^{-kL}} \right] \quad (9) \]

Case I. \( m \leq t \leq L/H \)

The total value of the interest payable during the first replenishment cycle is given by
\[ i_p = cl_m \int_m^L Y(t)e^{-kt} dt \]
\[ = cl_m \left[ -DL e^{-kt} + Dt e^{-kt} + D e^{-kt} \right]_m^L \]
\[ i_p = cl_m \left[ Q \left( \frac{e^{-km}}{k} \right) - D \left( \frac{me^{-km}}{k} + e^{-kL} - e^{-km} \right) \right] \quad \text{Where } Q = DL \quad (10) \]

The total present value of the interest payable over the time horizon \( H \) is given by
\[ Y_p = \sum_{j=0}^{n-1} i_p e^{-jL} = i_p \left[ \frac{1 - e^{-kH}}{1 - e^{-kL}} \right] \quad (11) \]

Also, the total value of the interest earned during the first replenishment cycle is
\[ E = cl \int_0^L Dte^{-kt} dt \]
A Cash Flow EOQ Inventory Model For Non-Deteriorating Items

\[ \dot{E} = c L D \left[ -t \frac{e^{-kt}}{k} - \frac{e^{-kt}}{k^2} \right] \]

Thus the present value of the total interest earned over the time horizon \( H \) is

\[ E_1 = \sum_{j=0}^{n-1} E_j e^{-jH} = E \left[ \frac{1 - e^{-kH}}{1 - e^{-kL}} \right] \]

Therefore, the total present value of the costs over the time horizon \( H \) is

\[ Z_1(L) = C_1 + C_2 + A + Y_p - E_1 \]

Substituting (7),(8),(9),(11) and (13) into (14), we have

\[ Z_1(L) = A_0 \left[ \frac{1 - e^{-kH}}{1 - e^{-kL}} \right] + c Q \left[ \frac{1 - e^{-kH}}{1 - e^{-kL}} \right] + h \left[ D \left( \frac{L}{k} + \frac{e^{kL} - 1}{k^2} - 1 \right) \right] \left[ \frac{1 - e^{-kH}}{1 - e^{-kL}} \right] + i_p \left[ \frac{1 - e^{-kH}}{1 - e^{-kL}} \right] \]

Case II. \( m > L = H/n \)

The interest earned in the first cycle is the interest during the time period \((0, L)\) plus the interest earned from the cash invested during the time period \((L, m)\) after the inventory is exhausted at time \( L \) and it is given by:

\[ E_2 = c L D \left[ -t \frac{e^{-kL}}{k} - \frac{e^{-kL}}{k^2} \right] \]

\[ E_2 = c L D \left[ -L \frac{e^{-kL}}{k} + \frac{1 - e^{-kH}}{k^2} \right] + (m - L) e^{-kT} \int_0^L R(t) dt \]

\[ E_2 = c L D \left[ -L \frac{e^{-kL}}{k} + \frac{1 - e^{-kH}}{k^2} \right] + (m - L) e^{-kL} DL \]

Hence, the present value of the total interest earned over the time horizon \( H \) is

\[ E_3 = \sum_{j=0}^{n-1} E_2 e^{-jH} = E_2 \left[ \frac{1 - e^{-kH}}{1 - e^{-kL}} \right] \]

Therefore, the total present value of the costs \( Z_2(n) \) over the time horizon \( H \) is

\[ Z_2(L) = C_1 + C_2 + A - E_3 \]

Substituting (7),(8),(9) and (17) into (18), we have

\[ Z_2(L) = A_0 \left[ \frac{1 - e^{-kH}}{1 - e^{-kL}} \right] + c Q \left[ \frac{1 - e^{-kH}}{1 - e^{-kL}} \right] + h \left[ D \left( \frac{L}{k} + \frac{e^{kL} - 1}{k^2} - 1 \right) \right] \left[ \frac{1 - e^{-kH}}{1 - e^{-kL}} \right] + i_p \left[ \frac{1 - e^{-kH}}{1 - e^{-kL}} \right] \]

OPTIMAL SOLUTION

To find the best value of \( L \) which optimizes (minimizes/maximizes) \( Z_1(L) \) and \( Z_2(L) \), we take the first derivative of \( Z_1(L) \) and \( Z_2(L) \) with respect to \( L \) and equate the result to zero. Also, to find if \( Z_1(L) \) or \( Z_2(L) \) is optimal (minimization/maximization) solution, we take the second derivative of \( Z_1(L) \) and \( Z_2(L) \) with respect to \( L \). If the result is less than zero is a maximization case and if the result is greater than zero is a minimization case.
Taking the first derivative of (18) with respect to \( L \), we have

\[
\frac{dZ_1}{dL} = \frac{dC_1}{dL} + \frac{dC_2}{dL} + \frac{dA}{dL} + \frac{dY_p}{dL} - \frac{dE_1}{dL}
\]

(20)

Now from (7), using Mclaurin’s expansion of exponential powers, we have

\[
C_1 = A_0 \left(1 - e^{-KH}\right) \left(1 - e^{-KL}\right)^{-1}
\]

\[
= A_0 \left[1 - \left(1 - KH + \frac{(KH)^2}{2!} - \frac{(KH)^3}{3!}\right)\left(1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!}\right)^{-1}\right]
\]

(21)

Taking the first and second derivative of (21) with respect \( L \) respectively, we have

\[
\frac{dC_1}{dL} = -A_0 \left[KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!}\right] \left[K - K^2L + \frac{K^3L^2}{2!}\right] \left[KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!}\right]^{-2}
\]

(22)

and

\[
\frac{d^2C_1}{dT^2} = -A_0 \left[KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!}\right] \left[-2 \left(K - K^2L + \frac{K^3L^2}{2!}\right) \left[KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!}\right]^{-3}\right] \left[K - K^2L + \frac{K^3L^2}{2!}\right]^{-2}
\]

(23)

From (8), we have

\[
C_2 = CDL \left(1 - e^{-KH}\right) \left(1 - e^{-KL}\right)^{-1}
\]

\[
= CDL \left[KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!}\right] \left[KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!}\right]^{-1}
\]

(24)

Taking the first and second derivative of (24) with respect \( L \) respectively, we have

\[
\frac{dC_2}{dL} = CD \left[KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!}\right] \left[KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!}\right]^{-1}
\]

\[
- L \left[KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!}\right]^{-2} \left[K - K^2L + \frac{K^3L^2}{2!}\right]
\]

(25)

and
\[
\frac{d^2 C_2}{dT^2} = CD \left( KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!} \right) \left[ -2 \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right)^2 \left( K - K^2 L + \frac{K^3 L}{2!} \right) \\
+2L \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right)^3 \left( K - K^2 L + \frac{K^3 L}{2!} \right)^2 \\
- L \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right)^2 \left( -K^2 + K^3 L \right) \right]
\]

From (9), we get
\[
A = \frac{D}{K^2} \left( KL + e^{-KL} - 1 \right) \left( 1 - e^{-KL} \right)^{-1}
\]

\[
A = \frac{D}{K^2} \left[ KL + 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} - 1 \right] \left[ -1 \left( 1 - KL + \frac{(KH)^2}{2!} - \frac{(KH)^3}{3!} \right) \right] \left[ 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right]^{-1}
\]

Taking the first and second derivative of (27) with respect \( L \) respectively, we have
\[
\frac{dA}{dL} = \frac{D}{K^2} \left( KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!} \right) \left[ \left( K^2 - K^3 L \right) \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right) \right]^{-1}
\]

\[
- \left( KL - \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right) \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right) \left( K - K^2 L + \frac{K^3 L}{2!} \right) \]

and
\[
\frac{d^2 A}{dL^2} = \frac{D}{K^2} \left( KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!} \right) \left[ \left( K^2 - K^3 L \right) \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right) \right]^{-1}
\]

\[
-2 \left( K^2 - \frac{K^3 L}{2!} \right) \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right)^2 \left( K - K^2 L + \frac{K^3 L}{2!} \right)
\]

\[
+2 \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right) \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right)^3 \left( K - K^2 L + \frac{K^3 L}{2!} \right)^2
\]

\[
- \left( KL - \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right) \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right)^2 \left( -K^2 + K^3 L \right)
\]

From (13), we get
\[ Y_p = C_l \frac{D}{K^2} \left( KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!} \right) \left[ \begin{array}{c}
-KL - \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \\
+Km - \frac{(Km)^2}{2!} + \frac{(Km)^3}{3!} \\
+K (L - m) \left( 1 - Km + \frac{(Km)^2}{2!} - \frac{(Km)^3}{3!} \right)
\end{array} \right] \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right) \]

Taking the first and second derivative of (30) with respect to \( L \) respectively, we have

\[ \frac{dY_p}{dL} = C_l \frac{D}{K^2} \left( KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!} \right) \left[ \begin{array}{c}
-KL - \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \\
+Km - \frac{(Km)^2}{2!} + \frac{(Km)^3}{3!} \\
+K (L - m) \left( 1 - Km + \frac{(Km)^2}{2!} - \frac{(Km)^3}{3!} \right)
\end{array} \right] \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right)^{-1} \left( K - K^2 L + K^3 L^2 \right) \]

and
From (16), we have

\[
E_1 = C_l \left( KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!} \right) \left[ \frac{D}{K} \left( L \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right) + \frac{D}{K^2} \left( KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right) \right] \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right)^{-1}
\]

Taking the first and second derivative of (33) with respect to \( L \) respectively, we have
\[
\frac{dE}{dL} = C_l\left(KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!}\right) \left[\frac{D}{K}\left(1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!}\right)\right] \\
- C_l\left(KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!}\right) \left[\frac{D}{K}\left(L - KL + \frac{K^3L^3}{2!}\right)\right] + \frac{D}{K^2}\left(K + K^2L - \frac{K^3L^2}{2!}\right)
\]

and

\[
\frac{d^2E}{dL^2} = C_l\left(KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!}\right) \left[\frac{-2D}{K}\left(-K + K^2L - \frac{K^3L^3}{2!}\right)\right] + \frac{D}{K}\left(L - KL + \frac{K^3L^3}{2!}\right) + \frac{D}{K^2}\left(K + K^2L - \frac{K^3L^2}{2!}\right) \\
- \frac{D}{K}\left(L - KL + \frac{K^3L^3}{2!}\right) - \frac{D}{K^2}\left(KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!}\right)
\]

From (21), we get
A Cash Flow EOQ Inventory Model For Non-Deteriorating Items

\[ E_3 = C_l D \left( KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!} \right) \]

\[
\left. \begin{array}{c}
- \frac{L}{K} \left( 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right) \\
+ \frac{1}{K^2} \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right) \\
+ (m - L)L \left( 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right)
\end{array} \right\}
\]

Taking the first and second derivative of (36) with respect \( L \) respectively, we have

\[
\frac{dE_3}{dL} = C_l D \left( KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!} \right) \]

\[
\left. \begin{array}{c}
- \frac{1}{K} \left( 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right) \\
- \frac{L}{K} \left( K + K^2 L - \frac{K L^2}{2!} \right) \\
+ \frac{1}{K^2} \left( K - K^2 L + \frac{K L^2}{2!} \right) \\
- L \left( 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right)
\end{array} \right\} \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right)^{-1}
\]

\[
\left. \begin{array}{c}
+ \frac{1}{K^2} \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right) \\
+ (m - L)L \left( 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right)
\end{array} \right\}
\]

and

\[
\left. \begin{array}{c}
- \frac{L}{K} \left( 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right) \\
+ \frac{1}{K^2} \left( K - K^2 L + \frac{K L^2}{2!} \right) \\
+ (m - L)L \left( 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right)
\end{array} \right\}\left( K - K^2 L + \frac{K L^2}{2!} \right)^2 \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right)^{-2}
\]

(37)
\[
\begin{align*}
\frac{d^2 E_1}{dL^2} &= Cl\cdot D\left( KH - \frac{(KH)^2}{2!} + \frac{(KH)^3}{3!} \right) \\
&= \begin{cases}
\frac{2}{K} \left( -K + K^2 L - \frac{K^3 L^2}{2!} \right) - \frac{L}{K} \left( K^2 - K'L \right) \\
+ \frac{1}{K^2} \left( -K^2 + K'L \right) - 2 \left( 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right) \\
+ 2(m - 2L) \left( -K + K^2 L - \frac{K^3 L^2}{2!} \right) + (m - L) \left( K^2 - K'L \right)
\end{cases} \\
&= \begin{cases}
\frac{1}{K} \left( 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right) - L \left( K^2 - K'L \right) \\
+ \frac{1}{K^2} \left( K - K^2 L + \frac{K^3 L^2}{2!} \right) - L \left( 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right) \\
+ (m - L) \left( 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right) \\
+ (m - L) L \left( -K + K^2 L - \frac{K^3 L^2}{2!} \right)
\end{cases}
\end{align*}
\]

Substituting (22), (25), (28), (31) and (34) into (20), and setting the result to zero, we have

(38)
A Cash Flow EOQ Inventory Model For Non-Deteriorating Items

\[ -A_0 \left[ K - K^2 L + \frac{K^3 L^2}{2!} \right] \left( KL - \left(\frac{KL}{2!}\right)^2 + \left(\frac{KL}{3!}\right)^3 \right)^{-1} \]

\[ + CD \left[ 1 - L \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right)^2 \left( K - K^2 L + \frac{K^3 L^2}{2!} \right) \right] \]

\[ + \frac{D}{K^2} \left( K^2 L - \frac{K^3 L^2}{2!} \right) - \left( \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right) \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right)^{-1} \left( K - K^2 L + \frac{K^3 L^2}{2!} \right) \]

\[ + CL \frac{D}{K^2} \left[ \left( -KL - \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right) + \left( KL - m \right) \left( 1 - km + \frac{(Km)^2}{2!} - \frac{(Km)^3}{3!} \right) \right] \left( K - K^2 L + \frac{K^3 L^2}{2!} \right) \]

\[ \left( K - K^2 L - \frac{K^3 L^2}{2!} \right) \left( KL - \left(\frac{KL}{2!}\right)^2 + \left(\frac{KL}{3!}\right)^3 \right)^{-1} \left( K - K^2 L + \frac{K^3 L^2}{2!} \right) = 0 \]

(39)

If other variables of (39) are known, then (39) can be used to determine the best (optimal) value of \( L \) which minimizes the total variable cost per unit time, provided that \( \frac{d^2 Z}{dL^2} > 0 \)

Now the second derivative of (15) with respect to \( L \) is

\[ \frac{d^2 Z}{dL^2} = \frac{d^2 C_1}{dL^2} + \frac{d^2 C_2}{dL^2} + \frac{d^2 A}{dL^2} + \frac{d^2 Y_p}{dL^2} - \frac{d^2 E_1}{dL^2} \]

(40)

Substituting (23), (26), (29), (32) and (35) into (40), we have

\[ \frac{d^2 Z}{dL^2} > 0 \]

(41)

Thus, \( Z \) is a minimization function.
Similarly for case II, taking the first derivative of (19) with respect to \( L \), we have
\[
\frac{dZ_2}{dL} = \frac{dC_1}{dL} + \frac{dC_2}{dL} + \frac{dA}{dL} + \frac{dY_p}{dL} - \frac{dE_3}{dL} \tag{42}
\]
Substituting (22), (25), (28), (31) and (37) into (42), and setting the result to zero, we have
\[
-A_0 \left[ K - K^2 L + \frac{K^3 L}{2!} \right] \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right)^{-1}
\]
\[
+CD \left[ 1 - L \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right)^2 \left( K - K^2 L + \frac{K^3 L}{2!} \right) \right]
\]
\[
+ \frac{D}{K^2} \left[ \left( K^2 L - \frac{K^3 L}{2!} \right) - \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right)^{-1} \left( K - K^2 L + \frac{K^3 L}{2!} \right) \right]
\]
\[
+ \frac{D}{K} \left[ \left( 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right) - L \left( -K + K^2 L - \frac{K^3 L}{2!} \right) \right]
\]
\[
- \frac{1}{K} \left( 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right) \left( -K + K^2 L - \frac{K^3 L}{2!} \right)
\]
\[
+ \frac{1}{K^2} \left( K - K^2 L + \frac{K^3 L}{2!} \right) - L \left( 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right)
\]
\[
+ (m - L) \left( 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right) - (m - L) L \left( -K + K^2 L - \frac{K^3 L}{2!} \right)
\]
\[
- \frac{L}{K} \left( 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right) \left( -K + K^2 L - \frac{K^3 L}{2!} \right)
\]
\[
+ \frac{1}{K^2} \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right) \left( K - K^2 L + \frac{K^3 L}{2!} \right) \left( KL - \frac{(KL)^2}{2!} + \frac{(KL)^3}{3!} \right)^{-1}
\]
\[
+ (m - L) L \left( 1 - KL + \frac{(KL)^2}{2!} - \frac{(KL)^3}{3!} \right) \right] = 0
\]
If other variables of (43) are known, (43) can be used to determine the best (optimal) value of \( L \) which minimize the total variable cost per unit time, provided that

\[
\frac{d^2 Z}{dL^2} > 0
\]

Now, the second derivative of (19) with respect to \( T \) is

\[
\frac{d^2 Z}{dL^2} = \frac{d^2 C_1}{dL^2} + \frac{d^2 C_2}{dL^2} + \frac{d^2 A}{dL^2} + \frac{d^2 Y_p}{dL^2} + \frac{d^2 E_3}{dL^2}
\]

(44)

Substituting (23), (26), (29), (32) and (38) into (44), we have

\[
\frac{d^2 Z}{dL^2} > 0
\]

Thus, \( Z \) is a minimization function.

**NUMERICAL EXAMPLES**

The following data gives an example to illustrate the result of the model developed in this study: demand \((D) = 700\) unit, \(A_0 = \$80\) unit^{-1}, holding cost \((h) = 2.6/\)unit/order, unit cost an item \((c) = \$15\), net discount rate of inflation \((k) = 0.2/\)year, interest charged per naira in stocks per year by the supplier \((Y_c) = \$0.16\), interest earned per naira per year, \((Y_e) = \$0.14\), and the planning horizon, \(H \) is 5 year. The permissible delay in settling account, \( m = 70/365 \) years. The computational outcome is shown in Table 1. In the numerical example we see that the case I is optimal in credit policy. The minimum total present value of cost is obtained when the number of replenishment, \( n \), is 85. With 85 replenishment the optimal (minimum) cycle time is 0.233 year, the optimal (minimum) order quantity, \( Q = 163.01 \) units and the optimal (minimum) total present value of cost, \( Z = \$35233.518 \). This means that the marketing department, production department and finance department are in an enterprise jointly to find the policy. Therefore, the policy involves inventory financing and marketing issue. So, we investigate that this model is very important and valuable to the enterprise.

**Table 1: Table of numerical examples**

<table>
<thead>
<tr>
<th>Cases</th>
<th>Order no (n)</th>
<th>Cycle Time L</th>
<th>Order Quantity(Q) units</th>
<th>Total Cost Z(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>81</td>
<td>0.221917808</td>
<td>155.3424658</td>
<td>35235.03808</td>
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<td>82</td>
<td>0.224657534</td>
<td>157.260274</td>
<td>35233.9747</td>
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<td>159.178082</td>
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<td>161.095890</td>
<td>35233.2067</td>
</tr>
<tr>
<td></td>
<td>85*</td>
<td>0.232876712*</td>
<td>163.0136986*</td>
<td>35233.51787*</td>
</tr>
<tr>
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<td>35234.22438</td>
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<tr>
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<td>35238.72022</td>
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<td></td>
<td>95</td>
<td>0.260273973</td>
<td>182.191780</td>
<td>35928.34487</td>
</tr>
</tbody>
</table>

*OPTIMAL SOLUTION: Table 1 above shows the results of the two cases (case I and case II) of the numerical examples at different point to test the applicability of the model developed. The optimal option in credit policy and the optimal total present value of the costs is obtain when the number of replenishment, \( n \), is 85, order quantity, \( Q = 163.01 \) with a total present value of the cost, \( z = 35233.51787 \).

**SENSITIVITY ANALYSIS**

Taking the parameters at the optimal point, changes in decrease of 5%, 10% and an increase in 5%, 10% of the demand, \( D \), the
From table 2 above, it can be seen that the holding and ordering costs react inversely to each other. As the order size increases, the holding and ordering costs react inversely to each other. As the order size increases,
fewer orders are required, causing the ordering cost to decline, whereas the average amount of inventory on hand will increase, resulting in an increase in carrying costs.

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REFERENCES


