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Full Length Research Article

2-DIMENSIONAL POSITION ERROR BIAS ANALYSIS OF AN ANGLE OF ARRIVAL BASED TARGET LOCATING SYSTEM

[1]Abdulmalik Shehu Yaro, [2]Adamu Abubakar Saidu

[1]Department of Communications Engineering, Ahmadu Bello University, Zaria, Nigeria
[2]Department of Electrical Engineering, Ahmadu Bello University, Zaria, Nigeria

*Corresponding Author Email Address: [2]abubakaras@abu.edu.ng
[1]yaroabdulmalik@yahoo.com

ABSTRACT
An angle of arrival (AOA) based locating system determines the location of an emitting target using its emission detected at spatially deployed ground station (GS) with an angulation algorithm. The position estimation (PE) accuracy of the system depends on several factors one of which is the approach to the development of the angulation algorithm. For passive target locating, the closed-form angulation algorithm is used and has been known to introduce bias in the PE process. In this paper, a bias analysis of the closed-form angulation algorithm is carried out to determine its percentage in the overall position mean square error (MSE). The analysis is carried out using a three-GS triangular configuration at some randomly selected unmanned aerial vehicle (UAV) drone locations. Monte Carlo simulation result based on 200 realizations shows that the bias error introduced by the angulation algorithm in the overall position MSE is about 64%. With the knowledge of the bias percentage, the actual locations of the UAV drones within the AOA-based locating system coverage can be determined.

Keywords: Angle of arrival; Bias analysis; Position mean square error; Monte Carlo; Triangular configuration.

INTRODUCTION
An active or passive wireless positioning systems estimate the position of an emitting target using its emissions detected at spatially located ground station (GS)s (Duran *et al.*, 2012). An angle of arrival (AOA) based target locating system is an example of the wireless positioning system that estimates the position of an emitting target in two stages (Yaro, Sani, & Musa, 2017). The first stage of the position estimation (PE) process involves estimating the AOAs of the target emission detected at the antenna of the GSs (Malajner, Gleich and Planinšič, 2015). In the second stage which is the scope of this paper, the AOAs obtained in the first stage are used as input to an angulation algorithm to determine the location of the emitting target (Yaro et al., 2017; Sha'ameri, et. al 2017). Obtaining the target locating using the angulation algorithm involves finding the inverse matrix solution of a least square (LS) problem (Yaro and Sha'meri, 2016).

The angulation algorithm used in the second stage to estimated target location depends on the mode of operation of the AOA-based system namely active or passive mode (Pirzada, et al, 2013; Yaro et al., 2017). In an active mode which means the location of the target is known however, there is a need to track the target using the AOA-based system. In this regard, the open-form angulation algorithm is used. As for the passive locating mode, the target location is not known, and, in this scenario, the closed-form angulation algorithm is used. Passive target locating using the AOA-based system is the scope of this paper thus, the closed-form angulation algorithm is considered.

The closed-form angulation algorithm has been known to introduce bias in the PE process resulting to high PE error (Dogançay, 2006). This is due to the algebraic manipulation involved in its development. Apart of the bias error which is peculiar to only the closed-form angulation algorithm, another factor that result in high PE error value irrespectively of the angulation algorithm is the condition number value of the coefficient matrix used in finding the inverse matrix solution of the LS problem (Yaro, Sha'ameri and Kamel, 2017). The higher the condition number value, the higher the PE error obtained by the angulation algorithm (Golub and Van Loan, 2013). Most of the techniques developed to improve the PE accuracy of the close-form angulation algorithm are targeted at reducing the condition number value of the coefficient matrix(Bishop *et al.*, 2008; Azzouzi *et al.*, 2011; Yaro, Sha'ameri and Kamel, 2017; Yarima, Sha'ameri and Yaro, 2018) . Even though the PE error is reduced, the bias introduced by the angulation algorithm still exist in the reduced PE error (Yaro, Sha'ameri, & Kamel, 2018). Thus, a bias analysis on the angulation algorithm is carried out in this paper. This is to determine the percentage of the bias error introduced in the overall PE error. By knowing the amount of bias error, the actual PE error in locating the target is determined by subtracting the PE error due to bias from the overall PE error.

Closed-Form Angulation Algorithm Development
In this section of the paper, a brief description of the development of the three-GS based 2-dimensional (2-D) closed-form angulation algorithm to determine the instantaneous location of an emitting target given the AOA measurements is presented. It is assumed that each GS is equipped with an *m*-element circular array antenna for the AOA estimation.

Let θ_1, θ_2 and θ_3 be the AOAs of an emitting target at GSs labelled 1, 2, and 3 respectively. In practical application, the emission from the target is corrupted with noise which results in an AOA estimation error. By modelling the AOA error as a zero mean Gaussian random variable, the estimated AOAs $(\hat{\theta}_n)$ in degree at the GS labelled 1, 2, and 3 are respectively shown in Eq. (1), Eq. (2), Eq. (3).

$$\hat{\theta}_{1,} = \theta_1 + N\left(0, \sigma_1\right) \quad (1)$$

$$\hat{\theta}_{2,} = \theta_2 + N\left(0, \sigma_2\right) \quad (2)$$

$$\hat{\theta}_{3,} = \theta_3 + N\left(0, \sigma_3\right) \quad (3)$$

where: σ_1, σ_2, and σ_3 in Eq. (1), Eq. (2) and Eq. (3) are the AOA error standard deviation (SD)s at GS labelled 1, 2, and 3 respectively.

The AOA error SD is a function of the signal to noise ratio (SNR) of the received target emission at each GS and the mathematically relationship for an 8-element circular array antenna is shown in Eq. (4) (Griffin, et al., 2015).

$$\sigma_i = 1.979 \times e^{(-0.2815 \times SNR)} + 1.884$$
$$\text{for } i = [1,2,3] \quad (4)$$

Let $\mathbf{x}_{e,[km]} = (x, y)$ be the instantaneous location of the emitting target in kilometer while $\boldsymbol{s}_1 = (x_1, y_1)$, $\boldsymbol{s}_2 = (x_2, y_2)$ and $\boldsymbol{s}_3 = (x_3, y_3)$ are the locations of GS labelled 1, 2 and 3 respectively. The estimated AOAs in Eq. (1) to Eq. (3) are related to the emitting target's location through a line of bearing (LOS) as shown in Eq. (5) to Eq. (7).

$$y = x \tan\left(\hat{\theta}_1\right) + y_1 - x_1 \tan\left(\hat{\theta}_1\right) \quad (5)$$

$$y = x \tan\left(\hat{\theta}_2\right) + y_2 - x_2 \tan\left(\hat{\theta}_2\right) \quad (6)$$

$$y = x \tan\left(\hat{\theta}_3\right) + y_3 - x_3 \tan\left(\hat{\theta}_3\right) \quad (7)$$

The Eq. (5) to Eq. (7) can be expressed in matrix form as shown in Eq. (8).

$$\begin{bmatrix} 1 & -\tan\left(\hat{\theta}_1\right) \\ 1 & -\tan\left(\hat{\theta}_2\right) \\ 1 & -\tan\left(\hat{\theta}_3\right) \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} y_1 - x_1 \tan\left(\hat{\theta}_1\right) \\ y_2 - x_2 \tan\left(\hat{\theta}_2\right) \\ y_3 - x_3 \tan\left(\hat{\theta}_3\right) \end{bmatrix} \quad (8a)$$

$$\mathbf{A}\mathbf{x}_e = \mathbf{b} \quad (8b)$$

The only unknown in Eq. (8) is the emitting target location, $\mathbf{x}_e$ and can be obtained given the GS coordinates and estimated AOAs by finding the inverse matrix solution. Several techniques to obtain the matrix solution to Eq. (8) have been presented in literatures (Markovsky, Sima and Van Huffel, 2010; Golub and Van Loan, 2013; Ford, 2014), but the optimum technique is the use of Singular Value Decomposition (SVD) total least square (TLS) (Markovsky, Sima and Van Huffel, 2010). Let matrix **C** be a concatenation of matrix **A** and **b.** Taking the SVD of matrix **C** as shown in Eq. (9) (Markovsky, Sima and Van Huffel, 2010).

$$\mathbf{C} = [\mathbf{A}, \mathbf{b}] = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathbf{T}} = \sum_{i=0}^{n+1} u_i \sigma_i v_i^T \quad (9)$$

The solution to Eq. (8) using the SVD TLS technique in Eq. (9) is presented in Eq. (10).

$$\mathbf{x} = -\frac{1}{v_{(n+1,n+1)}} [v_{(1,n+1)}, v_{(2,n+1)}, \cdots, v_{(n,n+1)}]^T \quad (10)$$

Target locating systems usually display the locations of target in cylindrical coordinate system that is in range (R_h) and bearing (θ_h). Conversion from the cylindrical coordinate system to the rectangular coordinate system is done using Eq. (11).

$$x = R_h \times \cos\left(\theta_h\right) \quad (11a)$$

$$y = R_h \times \sin\left(\theta_h\right) \quad (11a)$$

where: R_h in km is the horizontal range of the emitting target from the centre of the GS configuration and θ_h in degree is the horizontal bearing of the target with respect to true north.

The rectangular and cylindrical coordinate systems to represent target location is used interchangeable in the rest of the paper.

Closed-Form Angulation Algorithm Bias Estimation

As earlier stated, the closed-form approach to the development of the angulation algorithm introduce bias error in the PE process. Thus, it is important to know the percentage of bias error introduced by the angulation algorithm in the overall position mean square error (MSE). Eq. (12) shows the mathematically expression of the position MSE based on *N*-realization Monte Carlo (MC) simulation (Yaro, Sha'ameri & Kamel, 2018).

$$PE_{MSE} = \frac{1}{N} \sum_{n=1}^{N} \left[\left(\hat{x}_n - x\right)^2 + \left(\hat{y}_n - y\right)^2 \right] \quad (12)$$

where: $(\hat{x}_n, \hat{y}_n)$ is the estimate target location at the *n*-th MC simulation realization while (x, y) is the known target location.

The variance in the PE error due the bias introduced by the angulation algorithm is shown in Eq. (13) (Yaro, Sha'ameri and Kamel, 2018).

$$\gamma^2 = \frac{1}{N} \left(\left\| \sum_{n}^{N} \left(\hat{x}_n - x\right) \right\|^2 + \left\| \sum_{n}^{N} \left(\hat{y}_n - y\right) \right\|^2 \right) \quad (13)$$

Lastly, the variance in the angulation algorithm PE error due to the AOA error in Eq. (1) to Eq. (3) presented in Eq. (14).

$$\sigma_{PE}^2 = PE_{MSE} - \gamma^2 \quad (14)$$

Using Eq. (12) to Eq. (14), the percentage of the PE error due to bias introduced by the angulation algorithm and that introduced by the AOA error are determined.

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SIMULATION RESULT AND DISCUSSION

In this section of the paper, the percentage of the bias error introduced by the angulation algorithm in the overall position MSE is determined at some randomly selected target locations. The emitting target considered in an unmanned aerial vehicle (UAV) drone with transmitter and receiver parameters as presented in Table 1. The telemetry downlink frequency of 2.4 GHz from the UAV drone is considered in locating the drone using the AOA-based system.

Table 1. UAV drone and GS parameters for bias analysis (Zubir *et al.*, 2008; Vergouw *et al.*, 2016)

Parameter		Values
UAV drone	Antenna gain	2 dBi
	Frequency	2.4 GHz
	Transmit Power	100mWatt
GS	Antenna gain	13 dBi
	Receiver sensitivity	-90 dBm

The locations of the randomly selected UAV drones in cylindrical coordinate system for the analysis are presented in Table 2.

Table 2. UAV drone locations for the bias analysis

Drone location	A	B	C	D	E	F	G	H
θ_h (°)	0		30		60		90	
R_h (m)	100	200	300	400	500	600	700	800

The configuration in which the GSs are deployed contributes to the overall PE accuracy of the AOA-based locating system. It has been proved that the triangular configuration has the best PE accuracy as it resulted in the least PE error (Chen, et al., 2006) thus, is adopted in this paper. Figure 1 show the three-GS triangular configuration with each GS about 250 m from the center of the configuration.

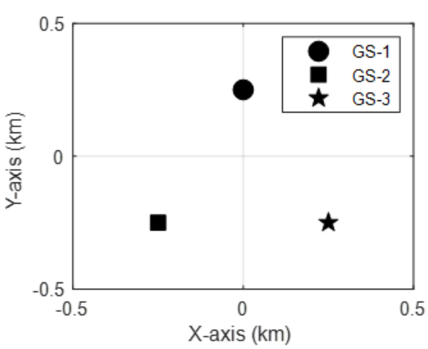

Figure 1: Three-GS Triangular configuration

Table 3 shows the position MSE and the PE error variance due to the bias introduced by the algorithm based on *N*=200 MC simulation realizations at each of the randomly selected UAV drone locations with coordinates as shown in Table 2. Both the PE_{MSE} and γ^2 varied with the UAV drone location. At UAV drone location A, the PE_{MSE} and γ^2 are 248 m² and 157 m² with an absolute difference of about 91 m² which corresponds to σ_{PE}^2. The percentage of γ^2 in PE_{MSE} obtained at location A is about 64%. This means that the percentage of the bias error variance in the overall position MSE by the angulation algorithm at UAV drone location A is 64% while that due to the AOA error is at about 36%.

Table 3. Variance in PE error due to angulation algorithm bias

Drone location	A	B	C	D	E	F	G	H
PE_{MSE} (m^2)	248	275	389	540	1823	2780	10140	15541
γ^2 (m^2)	157	175	247	343	1137	1735	6002	9040

Extending the analysis to UAV drone at locations B, C, D, E, F, G and H, the PE_{MSE} at each of the locations are 275 m², 389 m²,540 m², 1823 m², 2780 m², 10140 m², and 15541 m² respectively while the γ^2 are 175 m², 247 m², 343 m², 1137 m², 1735 m², 6002 m², and 9040 m², respectively. The percentage of γ^2 in PE_{MSE} at these locations are all about 64%. Thus, based on the selected UAV drone locations with coordinates presented in Table 2, the percentage bias error variance introduced by the angulation algorithm is on the average at about 64% while that of the AOA error is at about 36%. With knowledge of the percentage of bias error introduced by the angulation algorithm in the overall PE error, the actual position root mean square error (RMSE) due to the AOA error in locating the UAV drone can be determined which 36% of the position RMSE obtained is.

Figure 2 shows the position root mean square error (RMSE) obtained using the angulation algorithm based on *N*=200 MC simulation realization for UAV drones within the bearing range of 0⁰ to 359⁰ and horizontal range up to 1 km. Eq. (15) shows the mathematical expression for the position RMSE (Yaro *et al.*, 2017).

$$H_{rmse} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left[\left(\hat{x}_n - x\right)^2 + \left(\hat{y}_n - y\right)^2\right]} \qquad (15)$$

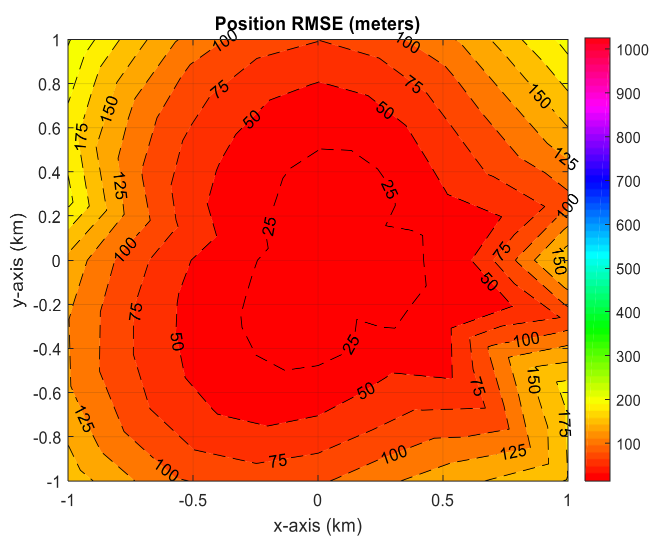

Figure 2: AOA-based target locating system position RMSE

The position RMSE increases with the UAV drone horizontal range and slightly invariant with the horizontal bearing. Table 4 shows the position RMSE at the randomly selected UAV drone locations with coordinate in Table 2. At UAV drone locations A, B, C, D, E, F, G and H, the position RMSEs are 16 m, 17 m, 20 m, 23 m, 43 m, 53 m, 101 m and 126 m respectively. Base on the bias analysis presented earlier, it was concluded that only 36% of the overall PE error is due to the actual AOA error. This means that the actual position RMSEs at these UAV drone locations are about ~6 m, ~6 m, ~7 m, ~8 m, ~19 m, ~37 m and ~ 47 m respectively. The remainder of the error obtained at each location is introduced by the angulation algorithm.

Table 3. Position RMSE at some randomly selected UAV drone locations

Drone location	A	B	C	D	E	F	G	H
$H_{rmse}\ (m)$	16	17	20	23	43	53	101	126

Conclusion

In this paper, the bias analysis and PE error of a closed-formed angulation algorithm is presented. This is to determine the percentage of the bias error introduced by the angulation algorithm in the overall position RMSE. The analysis is carried out at some randomly selected UAV drone locations using a three-GS configuration. Each of the GS is about 250 m from the center of the configuration. MC simulation result based on 200 realizations shows that percentage of the bias error variance introduced by the angulation algorithm in the overall position MSE is about 64%. With the understanding of the amount of bias introduced, the actual position of the UAV drone obtained by the AOA-based locating system within it coverage is determine.

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