Science World Journal Vol. 15(No 4) 2020
www.scienceworldjournal.org
ISSN: 1597-6343 (Online), ISSN: 2756-391X (Print)
Published by Faculty of Science, Kaduna State University

ON LOGISTIC GROWTH MODEL FOR FORECASTING NIGERIA'S POPULATION

Agog Nathan Samuel*, Bako Samuel Sunday, Bamanga Muhammad Ardo, Peter Magdaline, Byeli Usman

Department of Mathematical Sciences, Kaduna State University, P.M.B 2339, Tafawa Balewa Way, Kaduna.

*Corresponding Author's Email Address: nathan.agog@kasu.edu.ng

ABSTRACT
The population of Nigeria has been increasing steadily without a corresponding increase in resources required to cater for the population explosion. When the growth in resources is not keeping pace with the rapid growth in population, this poses a great threat to the nation in terms of social, environmental and economic development. This paper focused on the application of logistic model to estimate the population growth of Nigeria. The carrying capacity and vital coefficients of the population were estimated. The study revealed that the population of Nigeria grows at the rate of 3.0% per annum. The model was also employed to forecast for the Nigeria population. The study shows that the population of Nigeria will be approximately half of its carrying capacity of 485558039.4 in 58 years' time which corresponds to the year 2065.

Keywords: Logistic Model, Forecasting, Carrying Capacity, Vital Coefficients, Population Growth Rate.

1.0 INTRODUCTION

Population statistics of Nigeria in previous years reveals that it is increasing annually with about 2 to 3% growth rate. This is seen as one of the reasons for poor standard of living in Nigeria. Ashinze (2015) indicated that among other proffered solutions, arise the need to adequately model the Nigerian population that will effectively checkmate the unprecedented growth of the population with the resources available

Kerry *et al*. (2017) discuss the last census exercise conducted by the National Population Commission (NPC) in 2006. Nigeria's population was estimated to be at nearly one hundred and forty million. This is indeed an alarming figure for a country with distressed economy. The consequences of overpopulation is well known, with characteristics of socio-economic problems such as unemployment or underemployment, low level of per capita income, low standard of living, poverty, huge external debt burden and many more social vices. Apart from information on the stock of the country's population, it is essential to know the rate at which the population is changing, structurally and in the aggregate. For a country to develop socially and economically, it needs to ensure that its population growth do not overstretch its available resources to cater for the population. This is important not only for the present, but also for future planning and implementation of policies that will enhance economic growth and development.

Ogbeide and Ikpotokin (2010) conducted a study using logistic growth model to estimate the population of the people of Esan West Local Government Area of Edo State, where the growth rate was estimated to be 3.5%. According to Shilpa *et al*. (2014), uncontrolled human population growth has been a threat to the earth's resources and to the habitat themselves. Shilpa *et al.* (2014) estimated the population growth of India from 2009 to 2012 using the logistic model approach. The study gave a comparison of actual population of India and the estimated population using the logistic model, where they deduced an error equation based on the trend line for the specified time period. They concluded that the data fits well into the logistic growth model. Also, they concluded that it is necessary to estimate the population of India yearly rather than projecting it per census. The study also stated that it is important to understand the changing population trend for planning and implementation of policies related to population, environmental and economy of India.

Ofori *et al*. (2013) used the exponential and the logistic growth models to estimate the population growth of Ghana using data from 1960 to 2011. The result of their findings reveals that the exponential model predicted a growth rate of 3.15% per annum and the population predicted was 114.8207 million by 2050. The carrying capacity of Ghana was obtained using the logistic model and the vital coefficients *a* and *b* were 0.0523 and 7.7468×10^{-5}, respectively. This indicated that the logistic model reveals a growth rate of 5.23% and the population predicted was 341.2443 million by 2050.They conclude that the exponential model was the best for projection due to small value of mean absolute percentage error as compared to logistic model.

Sintayehu *et al.* (2016) used the Malthus's and the logistic growth models to estimate the population growth of Ethiopia using data from 1980 to 2016. The result from their findings indicated that the Malthus's population model predicted a growth rate of 2.9% per year while the logistic growth model predicted the carrying capacity of 19767334 and growth rate of 2.9% per annum. In their result, both exponential and logistic model match well with the growth rate estimated by International Data Base (IDB) for the past four years. The mean absolute percentage error was computed to be 0.64% for Malthus's growth model and 0.73% for the logistic growth model. From their research they conclude that the Malthus' population model seems to fit best to the original data since it has the smallest value of mean absolute percentage error.

Augustus *et al*. (2012) applied logistic equation in modeling the population growth and carrying capacity of Uganda. The method of least squares was used to estimate the population growth rate of Uganda. The values of the vital coefficients α and β were 0.0356 and 1.2056×10^{-10} respectively. Using the logistic model, it shows that the population growth rate of Uganda is 3.56% per annum and the population is expected to be 147,633,806 in the next 58 years from the year 2010, with a carrying capacity of 295,267,612. Minarul *et al.* (2012) used logistic population growth model for the projection of the population of Bangladesh for the year 2035. The results of their findings reveals that the carrying capacity was $K = 176771641.8$, and the vital coefficients *a* and *b* were 0.05 and $2.84000X10^{-10}$.

2.0 MATERIALS AND METHOD

In this study, secondary population data was taken from National Bureau of Statistics. The Logistic growth mathematical model was used to compute the carrying capacity as well as the projected population values of Nigeria.

2.1 The Logistic Growth Model

The logistic growth model was proposed by the Belgian Mathematician Verhulst (1838), which incorporates the idea of the carrying capacity. The population growth does not only depend on the population size but also how far this size is from its limit that is (maximum supportable population). This model is a modification of the Malthus model where population size is proportional to both the previous population and a new term.

$$\frac{dP}{dt} = \frac{a - bP(t)}{a} \tag{1}$$

Where a and b are the vital coefficients of the population. This term reveals how far the population is from its carrying capacity or maximum limit. Thus, as the population increases in size towards the carrying capacity, the value of the vital coefficients become very small and get to zero, providing the right information for the limiting value of the population growth. In this case, a modified equation which tends to limit the population growth is used to model the competition for available resources. The logistic growth model follows the assumption that each individual reproduces at a rate that deceases as a function of the population size. The modified equation is given as;

$$\frac{dP(t)}{dt} = \frac{aP(t)(a - bP(t))}{a}, \quad t_0 \le t \le t_1 \ ; P(t_0) = P_0 \tag{2}$$

Solving (2) and applying the initial conditions,

$$\frac{dP}{dt} = aP - bP^2 \tag{3}$$

Applying variables separation in (3) and integrating to obtain

$$\int \frac{1}{a}\left(\frac{1}{P} + \frac{b}{a - bP}\right) dP = \int dt$$

$$\frac{1}{a}\left(\ln P - \ln\left(a - bP\right)\right) = t + c \quad \text{at} \quad t = 0 \text{ and}$$

$$P = P_0 \tag{4}$$

$$c = \frac{1}{a}(\ln P_0 - \ln(a - bP_0)) \tag{5}$$

Solving for P by substituting (5) into (4) yields

$$P_t = \frac{\frac{a}{b}}{1 + \left(\frac{\frac{a}{b}}{P_0} - 1\right)e^{-at}} \tag{6}$$

Now taking the limit as $t \to \infty$ in (6)

$$P_{\max} = \lim_{t\to\infty} K = \frac{a}{b} (a \succ 0) \tag{7}$$

Putting $t = 1$ and $t = 2$, the values of P_1 and P_2 respectively, we obtain from (6) the following:

$$\frac{b}{a}\left(1 - e^{-a}\right) = \frac{1}{P_1} - \frac{e^{-a}}{P_0} \tag{8}$$

$$\frac{b}{a}\left(1 - e^{-2a}\right) = \frac{1}{P_2} - \frac{e^{-2a}}{P_0} \tag{9}$$

Dividing (9) by (8) yields

$$1 + e^{-a} = \frac{\left(\frac{1}{P_2} - \frac{e^{-2a}}{P_0}\right)}{\left(\frac{1}{P_1} - \frac{e^{-a}}{P_0}\right)} \tag{10}$$

Hence solving for e^{-a} to obtain

$$e^{-a} = \frac{P_0\left(P_2 - P_1\right)}{P_2(P_1 - P_0)} \tag{11}$$

Substituting equation (11) into equation (8) to obtain

$$\frac{b}{a} = \frac{P_1^2 - P_0P_2}{P_1(P_0P_1 - 2P_0P_1 + P_1P_2)} \tag{12}$$

Therefore, the limiting value of P is given by;

$$K = P_{\max} = \lim_{t\to\infty} P = \frac{a}{b} = \frac{P_1\left(P_0P_1 - 2P_0P_1 + P_1P_2\right)}{P_1^2 - P_0P_2} \tag{13}$$

3.0 DISCUSSION OF RESULTS

This section discusses results obtained for the carrying capacity, vital coefficients of the population, estimated logistic model for the population of Nigeria and finally the projected population of Nigeria using the estimated logistic model.

3.1 Estimating the Carrying Capacity of Nigeria

The carrying capacity is the maximum number of individuals that can be supported in an environment without experiencing decline in the capacity to support future generations within the area (Rees 1992).

Based on the population from the year 2006 in Table 1, let t=0, 1, 2 correspond to the years 2006, 2007 and 2008 respectively, where P_0, P_1, P_2 are 144858000, 148604000 and 152429000 respectively. Then the carrying capacity is obtained as follows;

$$K = \frac{P_1\left(P_0P_1 - 2P_0P_2 + P_1P_2\right)}{P_1^2 - P_0P_2}$$

$$K = \frac{2.513961211 \times 10^{21}}{2.588734 \times 10^{12}}$$

$$K = 971116078.7$$

The above result for the carrying capacity is the maximum pressure

Science World Journal Vol. 15(No 4) 2020
www.scienceworldjournal.org
ISSN: 1597-6343 (Online), ISSN: 2756-391X (Print)
Published by Faculty of Science, Kaduna State University

or load that Nigeria can conveniently accommodate before breaking down. A system breaks down when it can no longer accommodate the pressure of the population from the loads it is carrying.

3.2 Estimating the Vital Coefficients

The vital coefficients $(a \text{ and } b)$ reveal how far the population is from its carrying capacity or maximum limit. As the population increases in size towards the carrying capacity, the value of the vital coefficients become very small and get to zero, providing the right information for the limiting value of the population growth.

But, $$K = \frac{a}{b} = 971116078.7 \quad (14)$$

Using equation (11) we obtained $e^{-a} = 0.970372657$ and by taking the natural logarithm of both sides, we obtained

$$a = 0.030075098.$$

This result indicated that the predicted growth rate of Nigeria's population is approximately 3% per annum. Also, using equation (14) the value of the other vital coefficient b is obtained as;

$$b = \frac{0.030075098}{971116078.7} = 3.096962213 \times 10^{-11}$$

Let $t = 0$ coincide correspond to the year 2006 where the initial population $P_0 = 144858000$. The values of P_0, e^{-a} and $\frac{a}{b}$ are introduced into equation (6) to obtained;

$$P_t = \frac{971116078.7}{1 + (5.703917483) \times 0.970372657^t} \quad (15)$$

Equation (15) is the estimated logistic model for Nigeria. Table 1 shows the projected population values with the corresponding actual population values. The projected population values were computed using equation (15).

Table 1: Actual and Projected Population of Nigeria using Logistic Growth Model

Year	Population	Projected Population using Logistic Model
2006	144,858,000	144,858,000
2007	148,604,000	148,604,000.1
2008	152,429,000	152,429,000.1
2009	154,581,566	156,333,741
2010	159,608,173	160,318,921
2011	164,798,232	164,385,193.5
2012	170,157,060	168,533,163.6
2013	175,690,143	172,763,385.8
2014	181,403,148	177,076,360.8
2015	187,301,926	181,472,532.9
2016	193,392,517	185,952,287
2017		190,515,945.7
2018		195,163,766.4
2019		199,895,938.9
2020		204,712,581.9
2021		214,599,385
2022		219,669,404.7
2023		224,823,609.2
2024		230,061,723.7
2025		240,788,151.3

Hence the expected time for the population of Nigeria to be half of its carrying capacity $\left(\frac{a}{2b} = 485558039.35\right)$ can be obtained from equation (15), by substituting the value 48558039.35 for P_t,

$$P_t = \frac{971116078.7}{1 + (5.703917483) \times 0.970372657^t}$$

$$0.970372657^t = 0.1753181043$$

Taking logarithm of both side

$$t = \frac{\log 0.1753181043}{\log 0.970372657}$$

$$t \approx 58 \; years$$

Therefore, it is expected that the population of Nigeria would be half of its carrying capacity (48558039.35) in the year 2065.

4.0 Conclusion

In this study, we applied the logistic growth model in estimating the population growth of Nigeria. The carrying capacity including the vital coefficients that governed the population growth of Nigeria was determined. The results from the predictions revealed that the carrying capacity for the population of Nigeria is 971116078.7. The population growth rate was estimated from the vital coefficients which are $a = 0.030075098$ and $b = 3.096962213 \times 10^{-11}$ respectively. This vital statistics show that the population growth rate of Nigeria is 3.0% per annum which is similar to previous literatures that estimated a growth rate of between 2 to 3% annually. Also, the population of Nigeria will be approximately half of its carrying capacity (485558039.4) in 58years time, which corresponds to the year 2065.

The Vital coefficients should be assessed continually within short intervals of time in order to monitor variations in the population growth rate. Once there is good shelter and food supply, the vital coefficient b becomes smaller thereby increasing the carrying capacity of the country. Government at all levels should encourage and support large scale farming by subsidizing all the necessary farm inputs so as to increase food security.

REFERENCES

Ashinze A.N (2015). A Robust Model for the growth of the Nigerian Population. *IOSR Journal of Mathematics,* 11(6), pp 65-69

Augustus W., Epiphanie K. and Pacifique I. (2012). Mathematical Modeling of Uganda Population Growth. *Applied mathematical Sciences*, 6(84), pp 4155-4168.

Kerry C.C., Subeno T., Ezeora, J.N. and Okafor J.I., (2017). A Comparative Study of Mathematical and Statistical Models for Population Projection of Nigeria. *International Journal of Scientific and Engineering Research*, 8(2): pp 777-785.

Ogbeide E.M. and Ikpotokin O. (2010) Population Model of Esan West Local Government Area of Edo State, Nigeria. *Researcher*, 2(9), pp 27-30.

Rees W.E (1992). Ecological footprints and appropriated carrying capacity: what urban economics leaves out. *Environ urban* 4: pp 121-130.

Shilpah S. K., Shreenidhi R. K. and Suraj J. P. (2014). Analysis of Population Growth of India and Estimation for Future. *International Journal of Innovative Research in Science, Engineering and Technology*. 3(9), pp 15843-15850

Sintayehu A. M. (2016). Mathematical Model of Ethiopia's Population Growth. *Journal of Natural Sciences Research*, 6(17), pp 105-116

Ofori, T., Ephraim, T., and Nyarko, F., (2013). Mathematical Model of Ghana's Population Growth. *International Journal of Modern Management Sciences*, 2(2), pp 57-66

Minarul, H., Faruque A., Sayedul A. and Rashed K. (2012) Future Population Projection of Bangladesh by Growth Rate Modeling Using Logistic Population Model. *Annals of Pure and Applied Mathematics,* 1(2): pp 192-202

Verhulst, P.F., (1838). Notice sur la loique la population poursuit dans son Accroissement, Correspondance mathematique et physique, 10: pp 113-121