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Full Length Research Article

MATHEMATICAL MODELING OF FLUID FLOW AND TOTAL HEAT TRANSFER PROCESS IN WELLBORE

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ABSTRACT
Mathematical modelling of fluid flow and total heat transfer process in the wellbore is very important for predicting the actual situation in the realm. The wellbore consists of cement, tubing, casing, and for the total heat transfer, the effect of their respective temperatures on the flowing fluid and the surrounding earth must be investigated. The hot fluid from the reservoir moving up the wellbore had to pass through the tubing which is surrounded by casing and the cement and the surrounding earth formation and subsequently causes loss to the fluid temperature. Many studies about the wellbore consider that fluid temperature from bottom to top of the wellbore remain constant during the process and that heat transfer between the fluid and the surrounding earth temperature does not change resulting to inadequate optimization of wellbore function. This scenario if not properly studied will lead to inefficiency of optimising the wellbore or even premature closure of the wellbore. In this paper, a one-dimensional transient compressible model in the radial direction comprising the conservation of mass and momentum has been presented to investigate the behavior of the heat exchange between fluid temperature and the surrounding earth. Heat transfer equation was also developed to account for radii of tubing, casing and cement. The model was solved by flux vector splitting method of Steger Warming. The method allows the application of gas state equation which is best used in fluid temperature calculation and also account for heat exchange between fluid temperature and surrounding earth. It also allows investigation of the effect of wellbore temperature which is surrounded by casing and cement on the fluid temperature and can be extended to oil reservoir modelling especially in permafrost regions where geothermal gradient is significant. The result obtained shows that flowing fluid temperature drop toward the wellhead due to earth temperature effect on the flowing fluid. It can help gas production engineers in selecting types of pipes, casing and cement used in the wellbore construction.

Keywords: Wellbore, flowing fluid temperature, Earth temperature, Steger Warming Method.

INTRODUCTION

Mathematical model of fluid flow and total heat transfer process in the wellbore helps in investigating the behavior of heat exchange between earth temperature, flowing fluid temperature and the temperature of flow environment. Generally, research on fluid flow and temperature distribution in wellbores has been in place but no enough literature on the heat exchange between flowing fluid and the surrounding earth.

Recently, Jie et al., (2020), presented a wellbore temperature and pressure prediction model for wellbore temperature and pressure distribution along the well depth and compare the result obtained with actual wellbore data of Tarim X gas well and XX well in Southwest China and the calculation shows that the result of the model is less than 3% of the field measured data which verifies the accuracy of the model. Bo et al., (2021), reported that tubing leakage is one of the main reasons that cause annular pressure in HPHT gas wells. Their work further analyzed the relationship between leakage rate and sustained annular pressure and fluid temperature distribution. Hassan and Kabir, (2018), reported that the perfect balance between theory and practice to aid understanding of fluid flowing in the wellbore is to include probing pressure traverse in various multiphase fluid-flow situations, estimating flow rates from temperature data and translating off-bottom transient-pressure data to that at the datum depth. Mbaya and Amin, (2015), developed isothermal model for unsteady flow of gas in the producing well without the energy equation that account for the heat changes in the wellbore. Mbaya and Amin (2018) improved the work of Mbaya and Amin (2015) by considering energy equation for transient nature and heat transfer of fluid flow in a producing gas well. Farhan et al (2019) apply ANSYS Fluent to simulate the fluid flow and heat transfer in a weak wellbore of crude oil flowing upward in the tubing and gas (air) injected through holes to increase the wellbore production. Results show that crude oil velocity seems decreases downstream throughout the tubing. Liu et al (2013), presented a model for the determination of wellhead and bottom-hole pressure based on the principles of fluid dynamics considering fluid temperature to be constant neglecting the effect of the earth temperature on the temperature of the flowing fluid. Jiuping et al, (2013), developed a couple systems of partial differential equations for the variation of pressure, temperature, velocity and density at different time and depth in high pressure, high temperature well for two phases. Their solution considers splitting techniques with Eulerian Generalized Riemann Problems (GRP) schemes. Tong et al (2014) presented a transient non isothermal wellbore flow model for gas well testing. Their governing equation is based on depth and time dependent mass, momentum and gas state equation. Hassan and Kabir (2012) presented a unified approach for modelling heat transfer in various situations that result in physically sound solutions. Their work give much attention on many common elements, such as temperature profiles surrounding the wellbore and any series of resistances for the various elements in the wellbore Izgec et al (2007), proposed improvements to the previous analytic temperature models by developing a numerical differentiation scheme which removed the limitations imposed by the constant relaxation parameter assumption used in previous models. All available researchers did their work either in pipe flow transportation, analytical temperature distribution in oil and gas well but work is limited on the wellbore when considering the effect of the flow environment on flowing fluid to avoid inadequate optimization of wellbore function.

In this paper, a one-dimensional compressible model comprising conservation of mass and momentum has been presented to investigate the behavior of the heat exchange between fluid temperature, surrounding earth and the flow environment of the wellbore. It is solved by flux vector splitting method of Steger Warming. The method allows the application of gas state equation which account for heat exchange between fluid temperature and surrounding earth. It was chosen because it is unconditionally stable Toro E. F (2012). It also allows for investigating effect of wellbore diameter on the fluid and the temperature of the wellbore flow environment. The model can be extended to oil and gas reservoirs modelling especially in permafrost regions where geothermal gradient is significant.

Governing Equations

The equation governing gas flow and total heat transfer process in the wellbore which is surrounded by casing and cement are the Euler continuity, momentum and gas state equations in one dimension.

Equations of Continuity

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u}{\partial x}=0 \qquad (1)$$

Momentum

$$\frac{\partial G}{\partial t}+\frac{\partial(\rho u)}{\partial x}-\frac{\partial P}{\partial x}-\rho g\sin\theta-\frac{f\rho u^2}{2D}=0 \qquad (2)$$

Where ρ is the fluid density, u is fluid velocity, f is a friction factor, D is the well diameter, g is gravitation and θ is the inclination angle.

Gas State Equation

Wellbore is connected to the reservoir which consists of complex hydrocarbon mixture. An equation of state is an analytical expression which relates temperature and other gas properties. It has been proved to be used in phase behavior of pure substance and mixtures in the gas and liquid state since the period of Van Der Waals in 1873. Although at that time, the molecular structure of fluids had not been accepted by most physicists.

$$T=\frac{MP}{Z\rho R} \qquad (3)$$

where T is temperature of the flowing fluid depending on P pressure, M fluid mass $flow\ rate$ ρ fluid density, Z is gas compressibility and R is gas constant.

Equations (1) and (2) can be written in conservative as

$$\frac{\partial W}{\partial t}+\frac{\partial E(W)}{\partial x}=H(W) \qquad (4)$$

where

$$W=\begin{bmatrix} w_1 \\ w_2 \end{bmatrix},\quad E(W)=\begin{bmatrix} w_2 \\ \frac{w_2^2}{w_1}+a^2 w_1 \end{bmatrix},\quad H(W)=\begin{bmatrix} 0 \\ \frac{f w_2^2}{2Dw_1} \end{bmatrix} \qquad (5a,b,c)$$

In equation (5b) a^2 is the speed of sound given by, $a^2=\frac{\gamma P}{w_1}$,

w_1 is the gas density, w_2 is the mass flow rate and u is the axial velocity

Finite Scheme Method

As mentioned earlier, the Steger-Warming flux vector Splitting scheme method (FSM) has been considered in this work as the numerical scheme because literature has shown that it does not have the problem of numerical instability. In delta formulation, the finite difference form of the method is

$$-\left(\frac{\Delta t}{\Delta x}A_{j-1}^{+}\right)\Delta Q_{j-1}+\left(I+\frac{\Delta t}{\Delta x}\left(A_j^{+}-A_j^{-}\right)-\Delta t B_j\right)\Delta Q_j+\left(\frac{\Delta t}{\Delta x}A_{j+1}^{+}\right)\Delta Q_{j+1}$$

$$=\frac{\Delta t}{\Delta x}\left(E_j^{+}-E_{j-1}^{+}+E_{j+1}^{-}-E_j^{-}\right)+\Delta t H_j \qquad (6)$$

The subscript j indicates the spatial grid point while the superscript indicates the time level and

$$\Delta Q=Q^{n+1}-Q^{n} \qquad (7)$$

In equation (6) I is an identity matrix and A and B are Jacobian matrix defined by

$$A=\frac{\partial E}{\partial W},\qquad B=\frac{\partial H}{\partial W} \qquad (8a, b)$$

and A^{+}, A^{-} are positive and negative parts of the Jacobian matrix A defined as follows.

$$A^{+}=\begin{bmatrix} \frac{a^2-u^2}{2a} & \frac{u+a}{2a} \\ \frac{(u+a)^2(a-u)}{2a} & \frac{(u+a)^2}{2a} \end{bmatrix} A^{-}=\begin{bmatrix} \frac{u^2-a^2}{2a} & \frac{a-u}{2a} \\ \frac{(u+a)(a-u)^2}{2a} & -\frac{(a-u)^2}{2a} \end{bmatrix} \qquad (9)$$

In (6) also E^{+} and E^{-} are the positive and negative part of E defined as

$$E^{+}=\begin{bmatrix} \frac{w_1(u+a)}{2} \\ \frac{w_1(u+a)^2}{2} \end{bmatrix}\qquad E^{-}=\begin{bmatrix} \frac{w_1(u-a)}{2} \\ \frac{w_1(u-a)^2}{2} \end{bmatrix} \qquad (10)$$

Applying equation (6) to each grid point, a block tridiagonal system is formed. The equation is then solved at each time step which resulted to ΔQ and next Q can be calculated using equation (7).

Tridiagonal Decomposition

The Tridiagonal Decomposition which is sufficient for solving the coefficient of Q can be written in manageable form by defining the following

$$AM=-\frac{\Delta t}{\Delta x}A_{j-1}^{0+} \qquad (11a)$$

$$AA=\left[I+\frac{\Delta t}{\Delta x}\left(A_j^{0+}-A_j^{0-}\right)-\Delta t B_j\right] \qquad (11b)$$

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$$AP = \frac{\Delta t}{\Delta x} A_{j+1}^{0-} \quad (11c)$$

$$RHS = -\frac{\Delta t}{\Delta x}\left(E_j^{0+} - E_{j-1}^{0+} + E_{j+1}^{0-} - E_j^{0-}\right) + \Delta t H_j \quad (11d)$$

Thus Equation (6) can be expressed as

$$AM_i \Delta Q_{i-1} + AA_i \Delta Q_i + AP_i \Delta Q_{i+1} = RHS_i \quad (12)$$

Equation (11) form a system of tridiagonal that follows a system of computational domain. When Equation (13) is applied to each grid point i a system is formed. In this system the element of the coefficient matrix are themselves matrices. At various grid points we have;

$i = 2:$

$$AM_2 \Delta Q_1 + AA_2 \Delta Q_2 + AP_2 \Delta Q_3 = RHS_2 \quad (13)$$

$i = 3:$

$$AM_3 \Delta Q_2 + AA_3 \Delta Q_3 + AP_3 \Delta Q_4 = RHS_3 \quad (14)$$

$i = \mathrm{Im}\, m_2:$

$$AM_{\mathrm{Im}\,m2} \Delta Q_{\mathrm{Im}\,m3} + AA_{\mathrm{Im}\,m2} \Delta Q_{\mathrm{Im}\,m2} + AP_{\mathrm{Im}\,m2} \Delta Q_{\mathrm{Im}\,m1} = RHS_{\mathrm{Im}\,m2} \quad (15)$$

$i = \mathrm{Im}\, m_1:$

$$AM_{\mathrm{Im}\,m1} \Delta Q_{\mathrm{Im}\,m2} + AA_{\mathrm{Im}\,m1} \Delta Q_{\mathrm{Im}\,m1} + AP_{\mathrm{Im}\,m1} \Delta Q_{\mathrm{Im}} = RHS_{\mathrm{Im}\,m1} \quad (16)$$

In equation (11) the term ΔQ_1 is located at the inflow boundary between the wellbore and the reservoir. In (16) ΔQ_{Im} is at outflow leaving the wellbore to wellhead. Following this, a block tridiagonal matrix is produced and solved by tridiagonal solver.

$$\begin{bmatrix} AA_1 & AP_2 & & \\ AM_3 & AA_3 & AP_3 & \\ & AM_{\mathrm{Im}m2} & AA_{\mathrm{Im}m2} & AP_{IMM2} \\ & & AM_{\mathrm{Im}m1} & AA_{\mathrm{Im}m1} \end{bmatrix} \begin{bmatrix} \Delta Q_2 \\ \Delta Q_3 \\ \Delta Q_{\mathrm{Im}m2} \\ \Delta Q_{\mathrm{Im}m1} \end{bmatrix} = \begin{bmatrix} RHS_2 - A_{m2}\Delta Q_1 \\ RHS_3 \\ RHS_{\mathrm{Im}m2} \\ RHS_{\mathrm{Im}m1} - AP_{\mathrm{Im}m1}\Delta Q_{\mathrm{Im}} \end{bmatrix} \quad (17)$$

Flow Chart to the computational scheme is given in appendix B.

Steady State Equation

For effective application of the flux vector splitting method, the steady state solution of the governing equations is used as the boundary condition. The governing flow equations which consist of conservation of mass, momentum and gas state equation constitute the basis for all computation involving fluid flow in the wellbore and their application permit the calculation of changes in fluid temperature with distance.

Boundary Conditions

$$\frac{\partial \rho u(x,0)}{\partial x} = 0 \quad (18)$$

$$\frac{\partial \rho u(x,0)}{\partial x} + a^2 \frac{\partial \rho(x,0)}{\partial x} + \frac{f \rho u^2(x,0)}{2d} = -\rho g \mathrm{Cos}\theta \quad (19)$$

INITIAL CONDITIONS

Since the movement of the fluid in wellbore is from bottom to top, time steps is considered simultaneously with the length of the wellbore. The pressure is not also constant it changes due to the condition of the flow environment and the earth temperature.

$$\rho(0,t) = \rho_0(t),\ \frac{\partial u(0,t)}{\partial x} = u_0(t) \quad (20a,b)$$

$$T(0,t) = T_0(t),\ P(0,t) = P_0(t) \quad (21a,b)$$

where $\rho,$ are the inlet gas density, x, depth (m), T_f, is the temperature of the flowing fluid, is initial temperature of flow environment (tubing).

Compressibility Factor Model

The compression factor or the gas deviation factor, is a correction factor which describes the deviation of a real gas from ideal gas behaviour. It is an important parameter in determining the behaviour of flow in the wellbore when considering heat exchanges in the wellbore. In this work, Standing and Katz correlation which is most widely used in petroleum industries has been applied for calculating the Z- factor considering $P \leq 35$ Mpa

$$Z = 1 + \left(0.31506 - \frac{1.0467}{T_{pr}} - \frac{0.5783}{T_{pr}^3}\right)\rho_{pr} + \left(0.053 - \frac{0.6123}{T_{pr}}\right)\rho_{pr}^2 + 0.6815\frac{\rho_{pr}^2}{T_{pr}^3} \quad (22)$$

$$\rho_{pr} = \frac{0.27 P_{pr}}{Z T_{pr}},\ T_{pr} = \frac{T}{T_{pc}},\ P_{pr} = \frac{P}{P_{pc}} \quad (23)$$

T_{pc} is the critical temperature, P_{pc} is critical pressure T temperature and pressure *P*, of natural gas are all known.

3.4 Heat Transfer Model

Reservoir fluid is hot when compared with fluids outside. When this fluid enters a wellbore and begins to flow, it comes into contact with surrounding formations which has different temperature due to the flow environment and causing heat exchange between the temperature of the flowing fluid and the flow environment. The temperature model for temperature changes between the fluid considering tubing, casing, cement, and the surrounding formation can be derived as in G. Paul Wilhite (1966). He defined the rate of heat transfer between flowing fluid and the inside tubing, casing and cement during injection of hot water as;

$$Tubing = \frac{2\pi r K_{tub}(T_{ti} - T_{to})\Delta L}{\ln \frac{r_{to}}{r_{ti}}}$$

$$Ca\sin g = \frac{2\pi r K_{cas}(T_{ci} - T_{co})\Delta L}{lin \frac{r_{co}}{r_{ci}}}$$

$$Cement = \frac{2\pi r K_{cem}(T_{co} - T_h)\Delta L}{lin \frac{r_h}{r_{co}}}$$

Putting these together we obtain the heat transfer process equation which account for the heat change in the flowing fluid and the flow environment detail of derivation can found in Appendix A.

$$T_f = T_e + \frac{q}{2\pi\Delta x}\left[\frac{1}{r_{ti}h_f} + \frac{\log\left(\frac{r_{t0}}{r_{ti}}\right)}{k_e} + \frac{1}{r_{ci}h_{an}} + \frac{\log\left(\frac{r_{co}}{r_{ci}}\right)}{k_c} + \frac{\log\left(\frac{r_w}{r_{co}}\right)}{k_{cem}} + \frac{f(t_D)}{k_e}\right] \quad (24)$$

where h_f is thermal resistance of the earth formation, h_{an} thermal resistance of annulus, k_e thermal conductivity of earth, k_c thermal conductivity of casing, k_{cem} thermal conductivity of cement, $f(t_D)$ dimensionless function time r_{ti} inner radius of tubing, r_{to} outer radius of tubing, r_{ci} inner radius of casing, r_{co} outer radius of casing, r_w radius of the wellbore, T_f temperature of the flowing fluid, T_e initial undisturbed temperature of the surrounding earth.

$$f(t_D) = \begin{cases} 1.128\sqrt{t_D}\left(1-0.3\sqrt{t_D}\right) & t_D < 1.5 \\ \left(0.4063+0.5\log t_D\right)\left(1+\dfrac{0.6}{t_D}\right) & t_D > 1.5 \end{cases} \quad (25)$$

RESULTS AND DISCUSSION

The method was first compared with analytical solution at t=10s and t=25s and was observed to be in good agreed as shown in figure 1. It was also observed that the temperature of the flowing fluid drop with Casing, Cement and earth temperature respectively as in figure 2. Similarly figure 3 shows that the celerity wave remain slightly constant in the tubing but drop significantly in the casing to earth surrounding while figure 4 shows that the pressure drop significantly at the same condition.

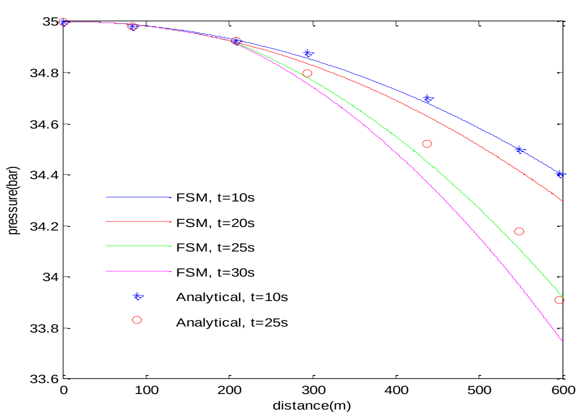

Figure 1: Comparison with Analytical solution at t=10s and t=25s

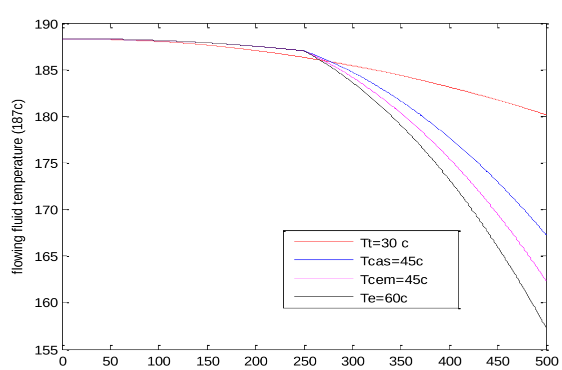

Figure 2: Temperature profile of the flowing fluid

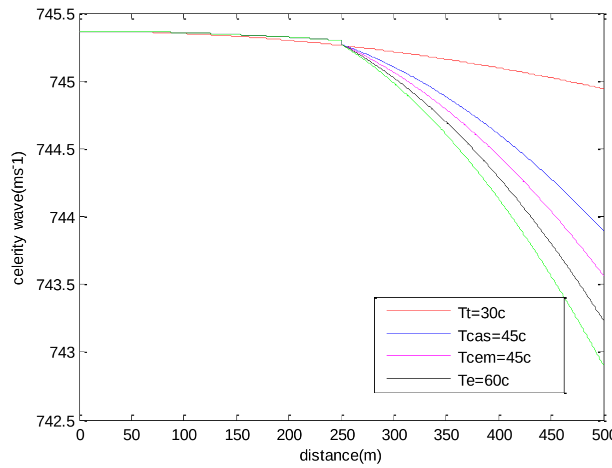

Figure 3: Profile of the celerity wave

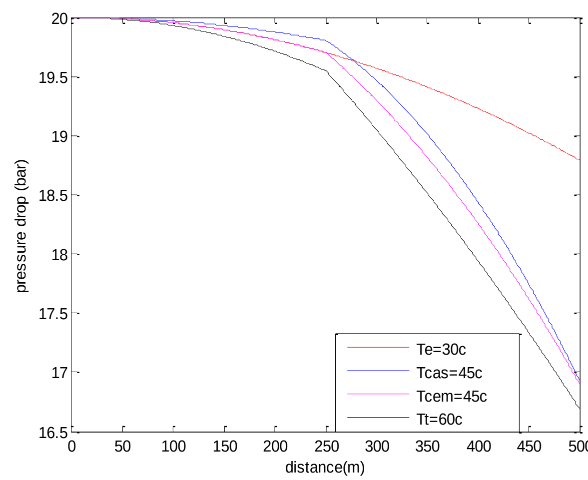

Figure 4: Pressure profile of the flowing fluid

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APPENDEX A: Heat Equation Procedure

The equation of total heat transfer in the wellbore (equation 24) can be obtain as following the rate of heat transfer between the fluid and the inside tubing wall as

$$Q = 2\pi r_{ti} h_f \left(T_f - T_{ti}\right)\Delta x \qquad A_1$$

h_f Represent heat coefficient for heat transfer considering surface area of tubing and the temperature difference between the flowing fluid and inside tubing wall $T_f - T_{ti}$.

Heat flow in tubing wall, casing wall and cement sheath occurs by conduction. It is assumed that heat flow through body is directly proportional to the temperature gradient in the medium and is represented by k_h where k_h is referred to as thermal conductivity in equation A2.

$$Q = 2\pi r k_h \frac{dT}{dr}\Delta x \qquad A_2$$

Keeping Q constant, A2 is integrated which gives equations A3 through A5

$$Tubing = \frac{2\pi r K_{tub}\left(T_{ti} - T_{to}\right)\Delta L}{\ln \frac{r_{to}}{r_{ti}}} \qquad A_3$$

$$Ca\sin g = \frac{2\pi r K_{cas}\left(T_{ci} - T_{co}\right)\Delta L}{lin \frac{r_{co}}{r_{ci}}} \qquad A_4$$

$$Cement = \frac{2\pi r K_{cem}\left(T_{co} - T_h\right)\Delta L}{lin \frac{r_h}{r_{co}}} \qquad A_5$$

Heat transfer in the wellbore consist of three categories, hot fluid near the tubing wall is less dense than the fluid in the centre and the annulus tends to raised. Defining heat transfer coefficient rate through the annulus in terms of the heat transfer coefficient h_c natural Convection and h_r natural radiation. Considering the outside tubing area $2\pi r_{to}\Delta x$ and the temperature difference between the outside tubing surface and inside casing surface. Thus

$$Q = 2\pi r_{to} h_f \left(h_c - h_r\right)\left(T_{to} - T_{ci}\right)\Delta x \qquad A_6$$

Now considering

$$T_f - T_h = \left(T_f - T_{ti}\right) + \left(T_{ti} - T_{to}\right) + \left(T_{to} - T_{ci}\right) + \left(T_{ci} - T_{co}\right) + \left(T_{co} - T_n\right) \qquad A_7$$

Assuming the heat flow in the well to be steady at particular time, the values of Q in equation A3 through A5 to be equal and solving for respective temperature difference in these equations and substituting them into A7 gives equation A8.

$$T_f = T_e + \frac{q}{2\pi\Delta x}\left[\frac{1}{r_{ti}h_f} + \frac{\log\left(\frac{r_{t0}}{r_{ti}}\right)}{k_e} + \frac{1}{r_{ci}h_{an}} + \frac{\log\left(\frac{r_{co}}{r_{ci}}\right)}{k_c} + \frac{\log\left(\frac{r_w}{r_{co}}\right)}{k_{cem}} + \frac{f\left(t_D\right)}{k_e}\right] \qquad A_8.$$

Equation A8 is the same as equation (24)

Appendix B: Computational Flow Chart